32.19 A $10cm$ solid copper ball initially at $400°C$ is placed in a large volume of $25°C$ liquid with unknown specific heat capacity. The average convective coefficient of heat transfer is $1000\frac{W}{m^2 \cdot K}$. How long does it take for the ball to cool to $100°C$?

A. $9\,seconds$

B. $16\,seconds$

C. $32\,seconds$

D. $95\,seconds$

The first step in a transient heat transfer problem is to determine whether the object can be treated as having Lumped Capacitance, in which case it is permissible to focus entirely on the external convective heat transfer to (or from) the object and assume that the internal conductive heat transfer within the object is relatively fast and thorough. If the lumped capancitance model does not apply, the heat transfer is said to be *distributed* through the object and the analysis becomes more involved. Check for the validity of using the lumped capacitance model by calculating the Biot number.

$$Bi = \frac{hV}{kA_s}$$

Pull out the ratio of the volume and surface area and calculate the result for a sphere.

$$\frac{V}{A_s} = \frac{\frac{4}{3}\pi r^3}{4\pi r^2} = \frac{r}{3} = \frac{d}{6}$$

Use the Properties of Metals table to obtain the conductivity of copper, and also make note of the densitiy and specific heat capacity for use in a subsequent step. Compute the Biot number.

$$Bi = \frac{\left(1000\frac{W}{m^2 \cdot K}\right)\left(\frac{0.1m}{6}\right)}{\left(403\frac{W}{m \cdot K}\right)} = 0.041 \ll 1$$

Since the Biot number is much less than 1, the lumped capacitance assumption is holds and it is valid to proceed without further considering internal conduction. Furthermore, since the volume of liquid is noted as being large, assume a Constant Fluid Temperature. The temperature variation of the sphere with time may be expressed with the formula below, where T is the current temperature of the sphere, $T_\infty$is the ambient temperature of the fluid (assumed to be constant), T_i is the initial temperature of the sphere, and β is further defined as below. The variable t is time, typically in $seconds$.

$$T - T_\infty = (T_i - T_\infty)\, e^{-\beta t}$$

$$\beta = \frac{hA_s}{\rho V c_p}$$

Solve for β. Recall that $\frac{V}{A_s} = \frac{d}{6}$. Note the inclusion of h, the convection coefficient, and the exclusion of k, the conductivity.

$$\beta = \frac{hA_s}{\rho V c_p} = \frac{\left(1000\frac{W}{m^2 \cdot K}\right)}{\left(8933\frac{kg}{m^3}\right)\left(\frac{0.1m}{6}\right)\left(389.4\frac{J}{kg \cdot K}\right)} = 0.017s^{-1}$$

Isolate the exponential term and calculate the ratio of the temperature differences.

$$e^{-\beta t} = \frac{T - T_\infty}{T_i - T_\infty} = \frac{100°C - 25°C}{400°C - 25°C} = 0.2$$

Take the natural log of both sides. Solve for time, t.

$$ln\left(e^{-\beta t}\right) = ln\left(0.2\right)$$

$$-\beta t = -1.61$$

$$\beta t = 1.61$$

$$t = \frac{1.61}{\beta} = \frac{1.61}{0.017s^{-1}} = 94.7s$$

Answer A